Modeling and Simulation of Heterogeneous Systems

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Description of Terminal Behavior

Heterogenous System (Principle)

Different physical domains

electrical subsystems

...
Heterogenous System (Principle)

Different description methods
Objective

- Simulation of complex systems
- Formulation of mathematical description as DAE-system
  \[ F(x, \dot{x}, p, t) = 0 \]
- Usage of existing simulation programs (Saber, ELDO, ...)
- Reusability of models
- Better understanding
Description of Terminal Behavior

Method

- Manual partitioning into subsystems

- Formulation of the terminal behavior of subsystems in a general mathematical form

- Implementation of the models with description languages like Mast, HDL-A, VHDL-AMS, Modelica, ...
Description of Terminal Behavior

Terminals of subsystems

flow quantity $i$
across quantity $v$
reference node

conservative pin
non-conservative pin
Description of the terminal behavior

- Division into dependent and independent terminal signals ($i_1$, $v_2$, $a_{out}$ resp. $v_1$, $i_2$, $a_{in}$)
- Usage of additional internal signal $s$

\[
i_1 = f_1(v_1, \dot{v}_1, i_2, \dot{i}_2, a_{in}, \dot{a}_{in}, s, \dot{s}, p, t)\]
\[
v_2 = f_2(v_1, \dot{v}_1, i_2, \dot{i}_2, a_{in}, \dot{a}_{in}, s, \dot{s}, p, t)\]
\[
a_{out} = f_3(v_1, \dot{v}_1, i_2, \dot{i}_2, a_{in}, \dot{a}_{in}, s, \dot{s}, p, t)\]
\[
0 = f_4(v_1, \dot{v}_1, i_2, \dot{i}_2, a_{in}, \dot{a}_{in}, s, \dot{s}, p, t)\]

signal values at $t$

$p$ parameters

$t$ time
Description of Terminal Behavior

General formulation

\[ d(t) = f_1 (u|t, s|t, p) \]
\[ 0 = f_2 (u|t, s|t, p) \]

- dependent terminal signal
- independent terminal signals
- parameters
- additional internal signals

\[ x : T \rightarrow \mathbb{R}^n \]
\[ x|t \quad \text{signal x until t} \]
Description methods
• Network models
• Control system blocks
• Bondgraphs

Derivation of terminal behavior
• Modeling methods based on System Theory
  - nonlinear static subsystems (RBF)
  - linear dynamic subsystems (recursive convolution)
• Physically based methods
  - analytically given methods
  - FEM-based methods

[ Simulator coupling ]
Network with different terminal types

Diode with selfheating effects

\[ \begin{align*}
    i_p &= i_D \\
    i_n &= -i_D \\
    i_T &= -(v_p - v_n) \cdot i_D
\end{align*} \]

with

\[
    i_D = I(v_T) \cdot \left( \frac{q \cdot \frac{v_p - v_n}{v_T}}{n \cdot k} \cdot e^{\frac{v_p - v_n}{v_T}} - 1 \right)
\]

\( p, n \) electrical terminals
flow quantities - currents \( i_p, i_n \)
across quantities - voltages \( v_p, v_n \)

\( T \) thermal pin
flow quantity- heat flow \( i_T \)

HDL-A Modell

ENTITY dio IS
    GENERIC ( IST0, T0, Pt, n, Eg, Rs : REAL );
    PIN ( p, n, T : ELECTRICAL; T : THERMAL );
END ENTITY dio;

ARCHITECTURE selfheating OF dio IS
    ...
END ARCHITECTURE selfheating;
CONTROL SYSTEM BLOCK

ENTITY PT1 IS
  GENERIC (K: REAL;
            T: REAL);
  PIN (I, O: NKN);
END ENTITY PT1;

ARCHITECTURE rt OF PT1 IS
  STATE S: ANALOG;
BEGIN
  RELATION
    PROCEDURAL FOR INIT =>
      K := 1.0;
      T := 1.0;
    PROCEDURAL FOR DC =>
      0.a := S;
    EQUATION (S) FOR DC =>
      0.0 == S - K*I.a;
    PROCEDURAL FOR TRANSIENT, AC =>
      0.a := S;
    EQUATION (S) FOR TRANSIENT, AC =>
      0.0 == T*ddt(Ua) + Ua - K*I.a;
  END RELATION;
END ARCHITECTURE rt;

Possibility to use control blocks together with networks.

Control system block

\[ \frac{K}{pT + 1} \]

\[ i \quad \rightarrow \quad o \]

\[ a_0 = s \]

\[ 0 = T \frac{ds}{dt} + s - K \cdot a_i \]
Different Description Methods

Bondgraph

R-bond

nonconservative terminals

input terminals

Series junction

\[
a_e = R \cdot a_f
\]

\[
a_f = \frac{1}{R} \cdot a_e
\]

\[
a_{f1} = s
\]

\[
a_{f2} = s
\]

\[
a_{f3} = s
\]

\[
0 = a_{e1} - a_{e2} - a_{e3}
\]
Nonlinear static subsystem

Approximation with radial basis function from given data points

\[ F(x) = \sum_{i=1}^{m} a_i \varphi(\|x - x_i\|) + \sum_{j=1}^{d} b_j x_j + b_0 \]

- \( F: \mathbb{R}^d \rightarrow \mathbb{R} \)
- \( \varphi \) is a given radial basis function (like \( \psi(r) = \sqrt{r^2 + c^2} \))
- \( a_i \) and \( b_i \) are determined from given data points
- \( m \) is less equal to the number of given data points
- an affin norm is used \( \|u\|_X^2 = m \cdot u^T V^TV^{-1} u \)
- Taylor series expansion if \( x = x_a + \Delta x \) near to the last argument \( x_a \)

Dew point sensor (example from S. Parodat: MARABU. Proc. Entwurf von Mikrosystemen, Dezember. 1997)
Usage of recursive convolution

- approximation of the characteristic function \( g(t) \) as a sum of exp-functions and \( \delta \)-terms interval by interval

\[
g_j(t) = v \cdot \text{Re}\left( \sum_{i=1}^{m} \alpha_i \cdot e^{\beta_i \cdot t} + k \cdot \delta(t - t) \right) \quad \text{for } t_{uj} \leq t < t_{oj}
\]

with \( v, k \in \mathbb{R} \) and \( \alpha_i, \beta_i \in \mathbb{C} \)

\[
g(t) = \sum_{j=1}^{n} g_j(t)
\]

- solution of the convolution \( a(t) = \int_{0}^{t} g(t - \tau) \cdot e(\tau) \, d\tau \)

by a recursive formulation

\[
a(t_n) = f(a(t_{n-1}), e(t_n), e(t_{n-1}))
\]
From FEM to network models

Energy: \( W = \ldots + W_i(v, \ldots) + W_j(v, \ldots) + W_k(v, \ldots) + \ldots \)

Minimum: \( \frac{\partial W}{\partial v} = \frac{\partial W_i}{\partial v} + \frac{\partial W_j}{\partial v} + \frac{\partial W_k}{\partial v} = 0 \) (KCL)
**Simple beam**

potential energy = deformation energy - energy of external forces

\[
W = \frac{1}{2} \int \int \int_V \sigma^T \varepsilon dV - E_1 v_{|1} - E_2 v_{|2}
\]

\[
\frac{\partial W}{\partial v_{|1}} = -\frac{E \cdot A}{L} (v_{|2} - v_{|1}) + E_{|1}
\]

\[
\frac{\partial W}{\partial v_{|2}} = -\frac{E \cdot A}{L} (v_{|1} - v_{|2}) + E_{|2}
\]
Framework

Principle:

Description of simple beam by 6-pol

netlist

* fixed points
v7 v20 0 0.0
v8 w20 0 0.0

* forces
i1 w7 0 -30.0
i2 w8 0 -30.0

Fixing of points by „voltage“ sources

Modeling of forces by „current“ sources
Micromechanical sensor

Inclusion of dynamic effects

\[
\begin{bmatrix}
F \\
T
\end{bmatrix} = -\left( \begin{bmatrix}
M \\
\end{bmatrix} \cdot \begin{bmatrix}
\ddot{\varphi} \\
\dot{\varphi}
\end{bmatrix} + D \cdot \begin{bmatrix}
\dot{\varphi} \\
\dot{\varphi}
\end{bmatrix} + S \cdot \begin{bmatrix}
\varphi
\end{bmatrix} \right)
\]

mass matrix  

damp matrix  

stiffness matrix  

displacements  

distortions  

torques  

excitation with force (components into all directions)
Modeling based on FEM

Inclusion of geometric nonlinearities

height 3 µm
width 15 µm
length 500 µm
density 2326 kg/m³
E-module 1,302 $10^{11}$ N/m²
shear module 79,62 $10^9$ N/m²

transfer characteristic

matrixes depend on displacements

\[
\begin{bmatrix}
F \\
T
\end{bmatrix} = -\left( M(v) \cdot \begin{bmatrix} \dot{v} \\\n\phi \end{bmatrix} + D(v) \cdot \begin{bmatrix} \ddot{v} \\
\ddot{\phi} \end{bmatrix} + S(v) \cdot \begin{bmatrix} v \\
\phi \end{bmatrix} \right)
\]
Conclusion

- Terminal behavior as basis for system simulation

- Unified approach to describe of the terminal behavior and its application to different description methods

- Derivation of the description of the terminal behavior using FEM-methods