Modeling acoustic transducer surface waves by Transmission Line Matrix method

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Summary
A model for the generation and propagation of acoustic and surface waves is implemented employing the Transmission Line Matrix (TLM) method with a locally reacting surface restoring boundary condition, which is a model for an array of capacitive ultrasound transducers. First test calculations clearly show the ability of the proposed model to capture acoustic and surface wave effects. The comparison with a reference solution derived with FE methods reveals that further refinements may be necessary although general agreement is achieved.

Keywords
TLM, surface waves, ultra sound, transducer array
1 Introduction

Micro-mechanic ultrasound transducer arrays are usually built on the basis of piezo electric bulk resonators. Alternative transducer principles, especially capacitive transducers are not employed because bulk resonators offer higher sound pressure levels, however with the drawback of a small bandwidth. In the last few years technologies were developed that allow to produce capacitive transducers with extremely small gap sizes (gap size means distance between the membrane and the counter electrode) on the basis of the standard BiCMOS process. This small gap enables high sound pressure levels and a large bandwidth at reasonable load voltages (about 10 V). Arrays of capacitive transducers consist of a large number of membranes next to each other which are set in motion by an alternating current. Experiments have shown that apart from sound waves in the plane of the transducer array surface waves may be generated and propagate, much like the gravity waves at the surface of a lake. This surface waves have long ring down times and may cause serious errors if the transducer array is used as a microphone. Therefore a method is needed to calculate the generation and propagation of these surface waves in order to develop and test strategies to suppress the unwanted surface effects.

The Finite Element Method (FEM) is a well-established tool for the numerical solution of partial differential equations. One disadvantage of the method results from the fact that the numerical effort tends to grow as the square of the number of nodes which discretize the problem. Thus the demand of a three-dimensional calculation grows with the power of six of a typical length. Unfortunately to resolve propagating waves the node density must not fall below a certain limit which prevents an enlargement of the element size far from the origin. This constraint makes the calculation of larger acoustic fields impossible. On the other hand, for wave propagation other numerical schemes exist which require computational resources directly proportional to the number of nodes. In this case the total cost of the computation is proportional to the power of three of a typical length of the problem in three dimensions.

One such scheme is the Transmission Line Matrix (TLM) method.

The theoretical foundations of the TLM method date back to the 40’s when Whinnery et al. and Kron developed equivalent circuit models made up of discrete elements for Maxwell’s equation to study wave guide models using network analyzers [1, 2, 3]. With the advent of more powerful computers the TLM-method was made popular by Johns and Beurle in 70s. Since then the method has been applied successfully to many problems which are described by wave or diffusion equations. The main difference between the TLM ansatz and standard schemes such as Finite Difference or FE methods is as follows:

Usually a physical problem is described precisely by a partial differential equation which subsequently is discretized. The solution to this equation is then approximated numerically. The TLM approach is to approximate the physical process (e.g. wave propagation) by discrete physical models, which can then be calculated exactly. This approach has several advantages. The physical models may be programmed intuitively, all discretization approximations are apparent, and the calculation may be parallelized easily by domain decomposition. The disadvantages are that TLM is best suited to equidistant nodes and fixed time steps, which are coupled to the spatial distance of the nodes. However, in recent years TLM was adapted for non-equidistant and curvilinear grids.

For complex technical devices such as acoustic transducer arrays it is required to compute several coupled physical effects (electro-magnetics, structural mechanics, fluid structure interaction) while large sound fields must be mapped. One approach is to couple TLM and FEM using FEM to calculate the structural and electromagnetic effects and using TLM to model the acoustic fluid. For this study a major problem was to simulate surface waves which are generated at the boundaries of the active area of a transducer array and which then propagate along the surface of the transducer array. These surface waves decay slowly and may cause significant deflections of the transducer membranes which are not wanted as they are not directly related to the acoustic field. As a first step the ability of the TLM-model to
simulate surfaces waves on a transducer array coupled to a fluid is studied. The next section describes the TLM theory and algorithm in more detail. Section 3 deals with the coupling of TLM and a simple model for the transducer array, which is to be replaced by FEM models in future. In the following section the current status of the work is reported by giving an example of a fluid/structure coupling.

2 TLM-Theory

The TLM scheme may be derived in two different ways. One way is to derive it as a special and simplified case of the more general Lattice Boltzmann schemes for the solution of the Navier-Stokes equations as was demonstrated by Chopard et al. [4]. Though this approach opens the way to inclusion of many nonlinear effects of sound propagation it is not very intuitive. Therefore another derivation shall be presented here which relies on a physical model of the field effects and was proposed by Kagawa for the acoustic field [5, 6].

The basic idea is to follow Huygens principle that states, that each point on a wavefront can be considered as a source of secondary wavelets, so that the propagation can be considered as a continuous process of scattering of elementary component waves. The superposition of all scattered (elementary) waves makes up the wave. This mechanism may be discretized as follows: The fluid is replaced by a network of pipes with equal diameters which are joint at fixed distances $\Delta x$ (see figure 1). The junctions are called nodes. Within the pipes acoustic pressure pulses may travel undisturbed at the speed of sound, i.e. no damping or boundary effects are considered. The diameter of the pipes is assumed to be much smaller than the shortest wavelength of interest so higher order pipe modes are neglected.

From this follows, that if a set of acoustic pressure pulses is released at the same time at arbitrary nodes, these pulses travel through the pipes and reach their neighboring nodes after a time interval of $\Delta t = \Delta x/c_0$ with $c_0$ being the speed of sound in the pipe. For 2 dimensions, the junction may be seen as a meeting of the pipe carrying the initial, incoming pressure pulse with three outgoing pipes. Thus the acoustic pressure pulse arriving at the node sees an apparent enlargement of the cross section by a factor of three, which means a corresponding change of the impedance. At this impedance mismatch the incoming pressure pulse is reflected with half of the initial amplitude and a reversal of sign, while it is transmitted into the other three pipes with half the initial amplitude and no change of sign (see figure 1).

From this point on the process of scattering and traveling is linear, superposition applies and the net result is the sum of the component effects. The algorithm described above can formally written as follows:

$$f_i(\vec{x} + \Delta \vec{x}_i, t + \Delta t) = S_{ij} f_j(\vec{x}, t)$$

with $f_i(\vec{x} + \Delta \vec{x}_i, t + \Delta t)$ representing the amplitude of the pressure pulse that travels from the node at position $\vec{x}$ to the node at position $\vec{x} + \Delta \vec{x}_i$ between times $t$ and $t + \Delta t$. $\Delta \vec{x}_i$ is the vector pointing to the nearest neighbor in the $i$-th direction, and $S_{ij}$ is the scattering matrix

$$S_{ij} = \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$

Because of the simple structure of the scattering matrix equation (1) may also be written as

$$f_i(\vec{x} + \Delta \vec{x}_i, t + \Delta t) = \frac{1}{2} \sum_j f_j(\vec{x}, t) - f_i(\vec{x}, t)$$

(2)
where \( i^* \) is defined such that \( \vec{x}_i = -\vec{x}_{i^*} \). The acoustic pressure \( p_{ak} \) and the velocity \( \vec{v} \) are calculated from \( f_i(\vec{x},t) \) applying the following formulae:

\[
p_{ak}(\vec{x},t) = \frac{1}{2} \sum_i f_i(\vec{x},t) \quad (3)
\]

\[
\vec{v}(\vec{x},t) = \frac{1}{\rho c^2} \sum_i \frac{\Delta \vec{x}_i}{\Delta t} f_i(\vec{x},t) \quad (4)
\]

This scheme is numerically equivalent to a second order Finite Difference scheme for the wave equation, which is shown as follows: The acoustic pressure at a position \( \vec{x} \) and time \( t + \Delta t \) is given as

\[
p_{ak}(\vec{x}, t + \Delta t) = \frac{1}{2} \sum_i f_i(\vec{x}, t + \Delta t)
\]

Repeated use of eq.\((3)\) yields

\[
p_{ak}(\vec{x}, t + \Delta t) = \frac{1}{2} \sum_i p_{ak}(\vec{x} + \Delta \vec{x}_i, t) - f_i(\vec{x} + \Delta \vec{x}_i, t)
\]

\[
= \frac{1}{2} \sum_i p_{ak}(\vec{x} + \Delta \vec{x}_i, t) - p_{ak}(\vec{x}, t - \Delta t) + f_i(\vec{x}, t - \Delta t)
\]

This may be rearranged to give

\[
p_{ak}(\vec{x}, t + \Delta t) - 2p_{ak}(\vec{x}, t) + p_{ak}(\vec{x}, t - \Delta t) = \frac{1}{2} \sum_i p_{ak}(\vec{x} + \Delta \vec{x}_i, t) - 2p_{ak}(\vec{x}, t)
\]

which equivalent to the corresponding Finite Difference equation if

\[
\frac{\Delta x}{\Delta t} = \sqrt{2}c
\]
speed of sound \( 1500 \text{ m/s} \)
fluid density \( 1000 \text{ kg/m}^3 \)
size of calculation area \( 4 \times 4 \text{ mm}^2 \)
number of nodes \( 800 \times 800 = 640000 \)
Spatial resolution \( \Delta x = 5 \mu \text{m} \)
temporal resolution \( \Delta t = 2.36 \cdot 10^{-9} \text{ s} \)
membrane compliance \( 12.25 \cdot 10^{13} \text{ Pa/m} \)
membrane mass \( 3 \cdot 10^{-7} \text{ kg/m}^2 \)
frequency of excitation \( 1 \text{ MHz} \)

Table 1: Parameters of the test calculation of surface waves.

This last result means that the speed of wave propagation \( c \) is given by the ratio of the spatial and temporal resolution. Note that the overall wave speed \( c \) is lower by the factor \( 1/\sqrt{2} \) than the speed of component pressure pulses traveling in the pipes, which is a feature of the method that is surprising when first encountered.

3 The ultrasound transducer array model

The ultrasound transducers to be modeled here are capacitive transducers made up of a flexible membranes above a fixed counter electrode. If a voltage is applied the membrane is attracted to the counter electrode until the electrostatic force is balanced by the restoring force of the membrane stiffness. Variations of the magnitude of the applied voltage lead to movements of the membrane which then acts as a sound generator for the fluid outside the transducer. The transducer is modeled as a simple mass-spring system where the spring with spring constant \( D \) provides the restoring force and the mass \( m \) corresponds to the membrane mass of the real transducer. From the fluid side the acoustic pressure \( p_{ak} \) calculated using eq.(3) acts on the membrane with area \( A \), also an electrostatic force \( F_{el} \) may be applied. The equation of motion for the membrane is

\[
m\ddot{s} = -p_{ak}A - Ds + F_{el}
\]

where \( s \) is the deflection of the membrane. This equation of motion is then solved employing the Newmark time integration scheme for every membrane and the resulting membrane velocity is applied as a boundary condition to the fluid by evaluation of eq.(4).

4 Results

As a first test a two dimensional setup is considered, which consists of water as the acoustic medium. The calculation area is \( 4 \times 4 \text{ mm}^2 \), three of the surrounding boundaries are rigid walls. The membrane array is placed along the bottom boundary. In this case one membrane model per TLM-node is applied and the membrane mass is chosen to be very small which corresponds to a free surface with locally reacting restoring force. Further details are given in table 1.

The first 80 membranes on the left are excited with a sinusoidal force with 1 MHz while all other membranes are allowed to move freely. After turning on the exciting force two effects clearly are observed:

- a sound wave is generated by the excited transducer and propagates into the fluid
- a surface wave is generated that spreads along the boundary of the fluid with lower propagation velocity than the sound wave
Figure 2: Pressure field at time $t = 2.83 \cdot 10^{-6}$ s after turning on the excitation force (2 MHz). White and black colors correspond to high and low pressure, respectively. The surface wave that propagates along the bottom boundary can clearly be seen to be slower (have shorter wavelength) than the sound waves, that spread into the medium.

Repeated experiments with other excitation frequencies show that the typical dispersion relations for sound waves $c = \lambda f$ and surface waves $c \propto \sqrt{\lambda}$ are satisfied by either wave type. Figure 2 shows the pressure field at the time $t = 2.83 \cdot 10^{-6}$ s after turning on the excitation force, here with a frequency of 2 MHz.

In order to compare the results quantitatively the same setup is also simulated using ANSYS. The spatial and temporal resolution is changed to $\Delta x = 20\mu m$ and $\Delta t = 2.83 \cdot 10^{-8}$ s to save calculation time.

Fig. 3 shows the membrane deflections as a function of position calculated with ANSYS and TLM at time $t = 2.83 \cdot 10^{-6}$ s after turning on the excitation force. Although the general shape agrees well for both calculations the amplitudes show differences of up to 20%. The reason for this differences is not yet understood.

Although its too early to compare calculation times it is noteworthy that the TLM-program takes about 2 minutes on 800x800 grid while ANSYS needs 20 minutes to complete on a 100x100 grid.

5 Summary and Conclusions

A model for the calculation of acoustic and surface waves in fluids is established on the basis of the Transmission Line Matrix (TLM) method. The free surface boundary condition is modeled by mass-spring-systems which are coupled to acoustic field by the acoustic pressure and velocity. First results clearly demonstrate the ability of this scheme to capture acoustic and surface wave propagation effects. Differences between the TLM solution and a reference solution calculated with the FE method indicate, that further checks or refinements may be necessary.

The high computational speed of the TLM-scheme allows 3d problems to be modeled. A 3d version of the TLM scheme was coded recently but the results have not yet been verified. First tests show that it’s possible to handle problems with up to 4 million nodes on a contemporary PC in reasonable time ($\approx 1$ s CPU time per model time step).
Figure 3: Deflection of the membranes as a function of position calculated with ANSYS (solid line) and TLM (dashed line) at $t = 2.83 \cdot 10^{-6}$ s after turning on the excitation force.

References


